Reduction Theory and Intelligent Control in Logical Networks

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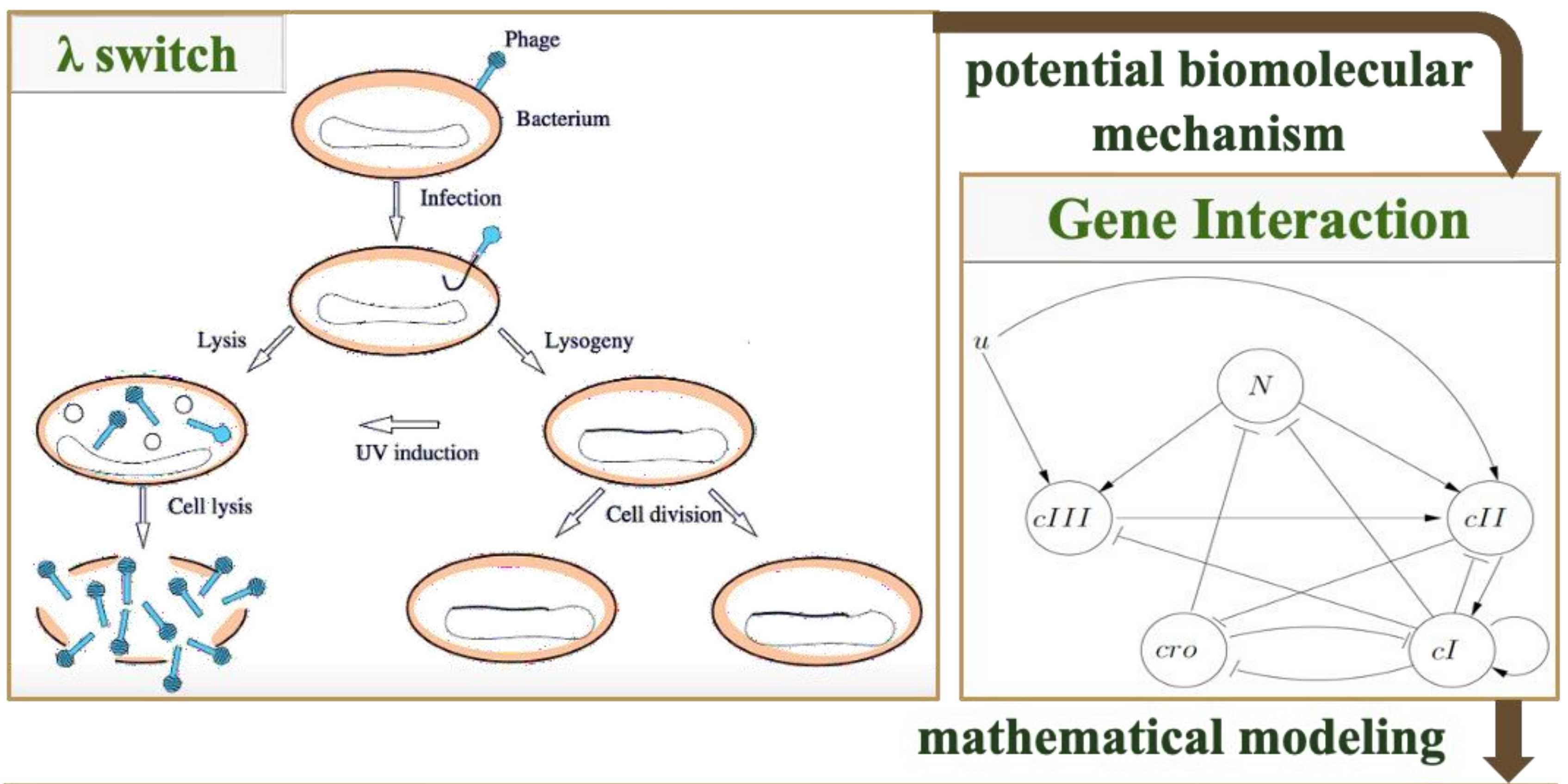
Logical Networks



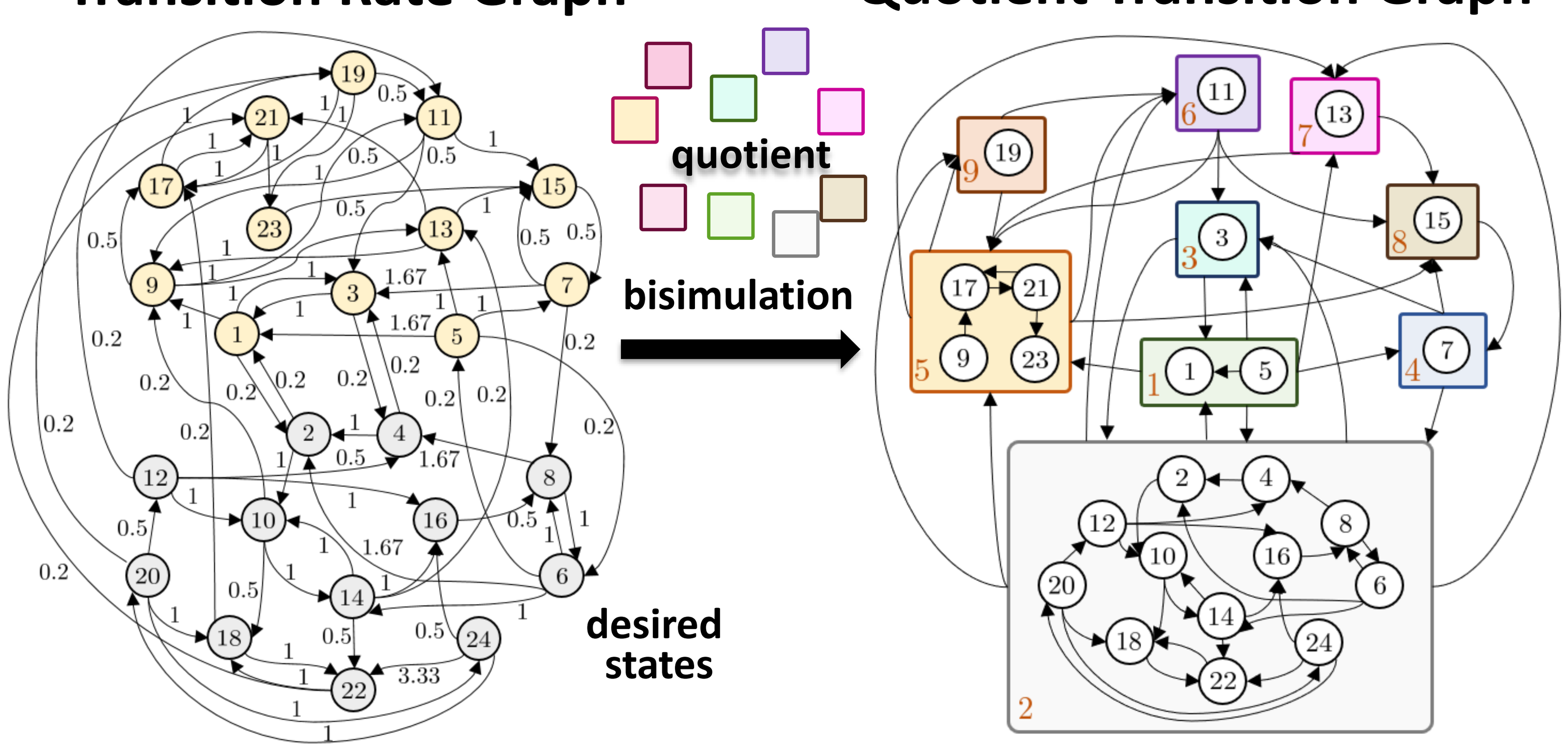
Logical Network

$$
\begin{aligned}
N(k+1) &= \neg cI(k) \wedge (\neg cro(k))\\
cI(k+1) &= \neg cro(k) \wedge (cI(k) \vee cII(k))\\
cII(k+1) &= \neg cI(k) \wedge u(k) \wedge (N(k) \vee cIII(k))\\
cIII(k+1) &= \neg cI(k) \wedge u(k) \wedge N(k)\\
cro(k+1) &= \neg cI(k) \wedge (\neg cII(k))
\end{aligned}
$$

Reduction Theory

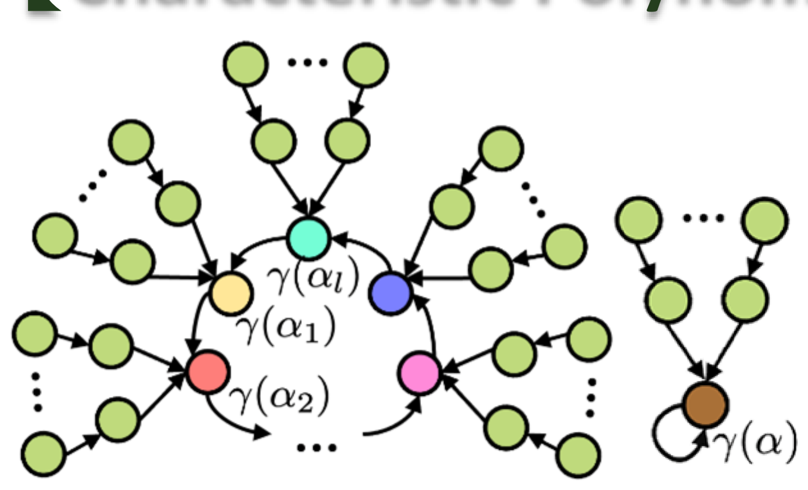
【Bisimulation-based Scale Reduction】

Transition Rate Graph — **Quotient Transition Graph**



[R1] L. Lin, J. Cao, J. Lam, L. Rutkowski, GM Dimirovski, and S. Zhu, **IEEE TAC**, 69(9): 5743-5758, 2024.

【Characteristic Polynomial Computation Reduction】



$$P_Q(\lambda) = \lambda^{(s+1)n-1}(\lambda - 1)$$

$$Q := \begin{bmatrix} F & 0_{n\times sn} \\ A_\star \otimes I_n & A_o \otimes I_n \end{bmatrix}$$

characteristic polynomial of network matrix

Approach	Time Complexity	Memory Storage
Existing works (network matrix)	$O(p^{3(s+1)n})$	$O(p^{(s+1)n})$
Transition graph	$O(p^{2(s+1)n})$	$O(p^{(s+1)n})$
Characteristic polynomial	$O((s+1)^3 n^3)$	$O((s+1)n)$

[R2] L. Lin, J. Cao, J. Lam, S. Zhu, S. Azuma, and L. Rutkowski., **IEEE TAC**, 69(7): 4718-4725, 2024.

Learning-Based Control Framework

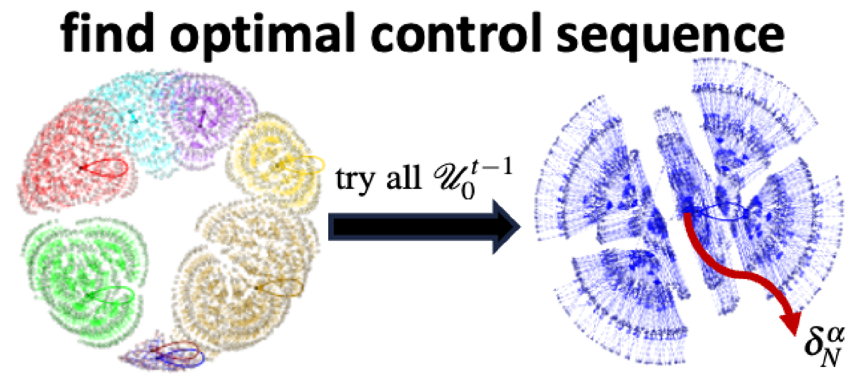
Algebraic State Space Model

Calculate stabilizability probability

Matrix powers sum

$$\max_{\mathscr{U}_0^{T-1}\subseteq\Delta_M} \mathrm{P}\Big\{\bigvee_{\iota=1}^{T} \mathbf{x}(\iota;\mathscr{U}_0^{\iota-1},\delta_N^i) \succeq \delta_N^\alpha\Big\}$$

Traversing to find optimal control sequence



Model-dependent Approach

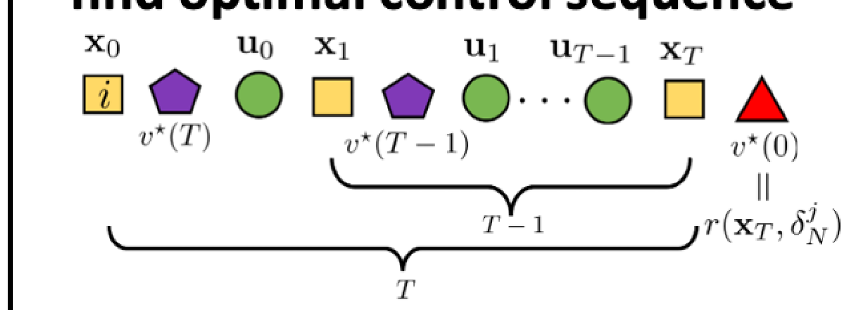
Markov Decision Process Model

Calculate reachability probability

Recursive equation

$$\max_{\mathbf{u}\in\Delta_M}\Big[\mathrm{E}\{r(T)\mid \delta_N^i,\mathbf{u}\} + \sum_{\varsigma\neq\alpha}\big((\Upsilon\mathbf{u})_{\varsigma,i}\cdot v^*(T-1,\delta_N^\varsigma)\big)\Big]$$

Iteratively find optimal control sequence

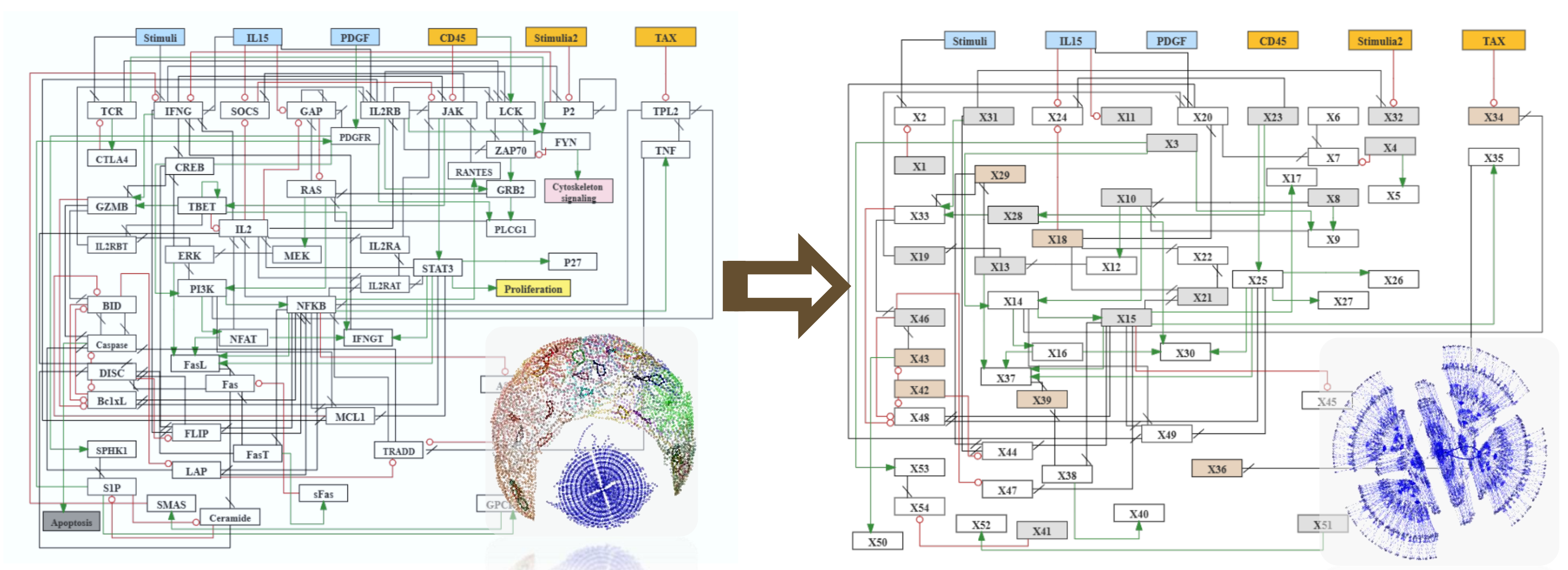


Model-free Approach

[R3] L. Lin, J. Lam, P. Shi, M. Ng, and H.-K. Lam, **IEEE TAC**, 70(6): 3889-3904, 2025, and **ICMLC 2023 (Lotfi Zadeh Best Paper Finalist)**.

Lightweight Control Strategies

➢ **Pinning Control Strategy:**

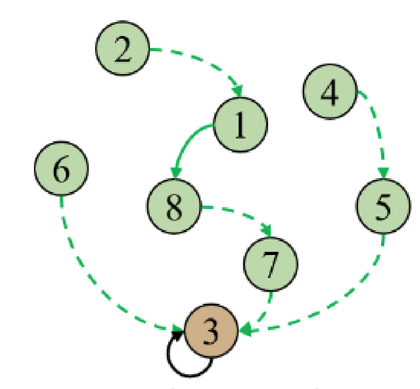


[R4] L. Lin, J. Lam, W.-K. Ching, Q. Qiu, L. Sun, and B. Min, **IEEE CYB**, 55(5): 2098-2109, 2025.

➢ **Optimal Triggering Control Strategy:**

$$\min_{\Lambda\subseteq\Delta_n} |\Lambda|$$

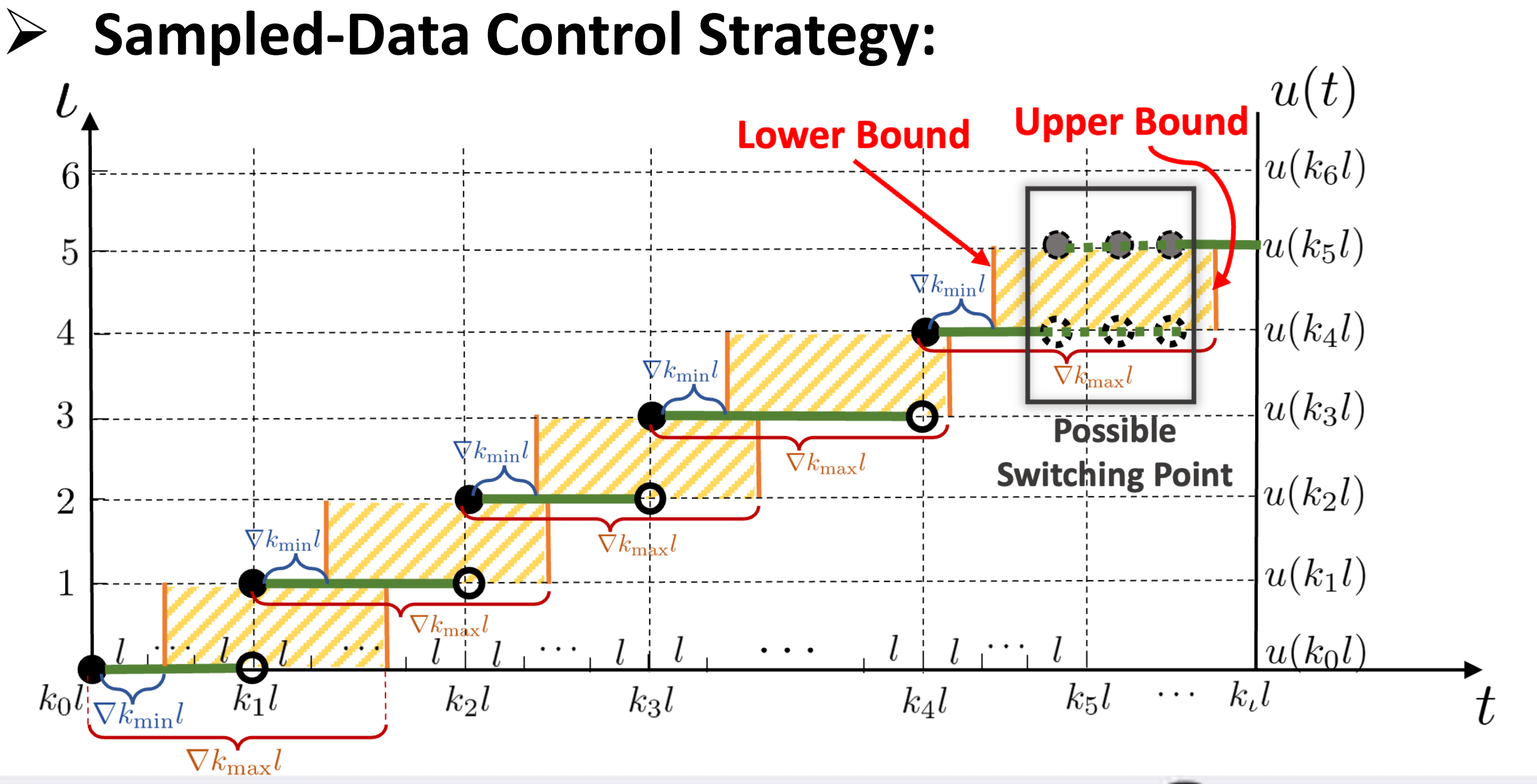
$$\begin{cases} \breve{A} := \mathbb{D}_{\Delta_n\backslash\Lambda}\{F\} \vee \mathbb{D}_\Lambda\Big\{\bigvee_{j=1}^{m} G^{(j)}\Big\}\\ \breve{A}\delta_n^\alpha = \delta_n^\alpha\\ \breve{A}^n = \delta_n^\alpha \mathbf{1}_n^\top \end{cases}$$



minimum-weight arborescence

[R5] L. Lin, M. Meng, Z. Shu, Z. Zhong, and J. Lam, **Automatica**, 177: 112290, 2025.

➢ **Sampled-Data Control Strategy:**



Feasible stabilizer set for sampled-data control: $\mathcal{D}_{[\nabla k_{\min},\nabla k_{\max}]}$

[R6] L. Lin, J. Lam, M. Meng, X. Xie, P. Li, D. Zhang, and P. Shi, **SICON**, 62(2): 1006-1033, 2024.